action and therefore the adiabatic surface temperature T_s' are not known but the calculation is relatively insensitive to the possible variation in these parameters. We shall assume $(T_s' - T_0)$ equal to 250°C corresponding to a heat of reaction in the solid phase of approximately 90 cal/g."

Experiment

We now have an experimental method for measuring the value T_s' to use in Adams' theory to calculate the condensed phase heat of reaction.

Briefly, the method suggested by Suh et al.[4] consists of raising the initial temperature of the propellant to various desired temperatures uniformly by passing hot air around it and measuring the burning rates. Figure 4 of Ref. 4 shows the experimental results for burning rate as a function of initial temperature. The asymptotic temperature at which the burning rate approaches infinity is found to be 145°C (290°F).

This is the minimum surface temperature at the solid surface with no heat feedback from the gas phase. The experimental arrangement indicates that if a piece of propellant is heated to 145°C, it is just about to ignite by itself and the slightest amount of energy which is added to it externally results in sudden ignition and rapid deflagration at a rate approaching infinity. This is therefore the temperature at which no heat feedback from the flame is necessary and it does burn at a rate approaching infinity just as Eq. (15) predicts that it should.

Discussion

It is important to remember that the temperature T_s' does not actually exist at the burning surface during the experimental procedure described herein and it may actually never exist at the surface of a steady-state burning propellant. We are not determining T_s' from a direct measurement of the surface temperature itself (which may be impossible with present technology) but asymptotically with an indirect measurement from the other end of the flame front which is much more reliable experimentally, and which logically does meet the requirement of no heat feedback from the gas phase.

The possibility of condensed phase reactions occurring during the experiments while the propellant sample is being heated to a desired initial temperature has been suggested. However, Ref. 2 indicates that at a heating rate of 20°C per minute, which is approximately the rate at which the propellant samples were usually heated, significant heat generation does not occur until temperatures are above 165°C.

Conclusions

The method presented in this paper provides a simple technique for isolating the condensed phase contribution to the over-all heat of reaction of a deflagrating double-base propellant. With M-2 type propellant for which T_s' is 145°C, $\bar{c}_p = 0.37$, and assuming an initial temperature of $T_0 = 25$°C we obtain

$$Q_r = 0.37(145 - 25) = 45 \text{ cal/g}$$

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Laminar Incompressible Flow Past a Circulation-Controlled Circular Lifting Rotor

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Introduction

In the flow past a lifting body, the lift predicted from potential flow considerations is usually not realized due to boundary-layer separation. One way to alleviate this problem is to introduce a tangential jet to postpone the onset of separation.

To study the problem of flow past a body with circulation control, the problem of a jet on a curved surface in the presence of an external stream must be considered. Numerical solutions for the laminar, two-dimensional, high Reynolds number case with no external pressure gradient have been obtained by Pai and Hsieh[1] and Kleinstein[2] for the freejet and straight wall jet, respectively. Schetz and Jannone[3] have considered the effect of an upstream boundary layer for the straight wall case.

Methods of solution have been presented for treating the turbulent flow past circulation-controlled rotors. Among these investigations are the circular cylinder of Dunham[4] and the elliptic cylinder of Kind.[5] The turbulent problem involves the introduction of much empirical information. It is for this reason that the laminar case is studied in the hope that important trends can be predicted that compare favorably with experimental observations.

Laminar Incompressible Jet on a Circular Cylinder

The flow configuration is shown in Fig. 1. θ is the angle measured from the forward stagnation point and the blowing slot is situated at $\theta = 90°$. The speeds of the uniform stream and the jet are U and U_j, respectively. The cylinder radius is R and the slot height is h.

The equations to be solved are the first-order boundary-layer equations of Prandtl for two-dimensional, laminar, incompressible flow. The outer inviscid flow is the potential flow solution for the cylinder without the jet. The equations are solved numerically using the implicit finite-difference scheme of Blottner and Flügge-Lotz.[6]

The jet strength can be varied by changing either U_j or h. Experimental results are usually correlated using the momentum coefficient $C_\mu = hU_j^2/2RU^2$ as the jet parameter. The suitability of this parameter will be investigated.

For now, the boundary layer upstream of the slot is not taken into account. Its effect will be studied later. The

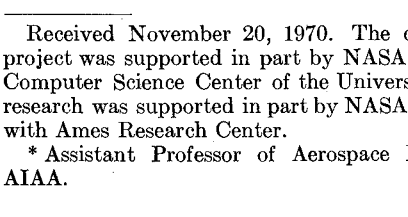

Fig. 1 Schematic representation of the flow.

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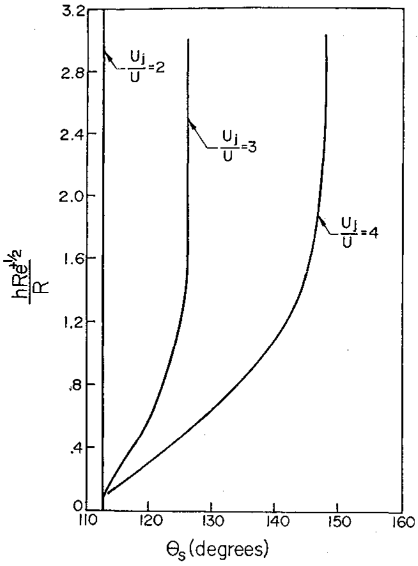

Fig. 2 Flow separation angle vs slot height for jet on circular cylinder.

initial axial velocity component u at the slot location is

$$u = U_j(O \leq y \leq h),\ u = 2U(h < y \leq H)$$

where y is measured normal to the cylinder surface and $H \gg \delta$, the boundary-layer thickness. h/R is varied from 0.1 $Re^{-1/2}$ to $3Re^{-1/2}$ where Re is the Reynolds number. $Re = UR/\nu$ and ν is the kinematic viscosity. U_j/U is varied from 2 to 4.

The angle at which the flow separates from the cylinder, θ_s, is plotted vs. h/R in Fig. 2. There are separate curves for each value of U_j/U. Each curve is seen to approach a constant θ_s as h/R is increased, and it is apparent that the parameter C_μ becomes inappropriate once this constant value is reached. Once the results lying on the constant portion of the curves are removed, the remaining results are plotted vs C_μ in Fig. 3 and reasonably good correlation is achieved. For large values of h/R, it appears that U_j/U is an appropriate jet parameter. On the basis of experimental results, Dunham[7] states that C_μ is the appropriate jet parameter for moderate values of h/R but that U_j/U is better for larger values.

To study the effect of the upstream boundary layer, the initial velocity profile for $y > h$ is taken as the boundary-layer profile at the slot location. Jet speeds of $3U$ and $4U$ are considered for a wide range of h/δ. The results for the wall shear stress C_f are shown in Fig. 4. They are similar qualitatively to those of Schetz and Jannone[3] for the straight wall case. The deviation from the no upstream boundary-layer case ($\delta = 0$) is most significant for small values of h/δ. As h approaches δ, the effect of the upstream boundary layer becomes negligible except perhaps in the immediate neighborhood of the separation point. The effect is also most significant for the smaller values of U_j/U.

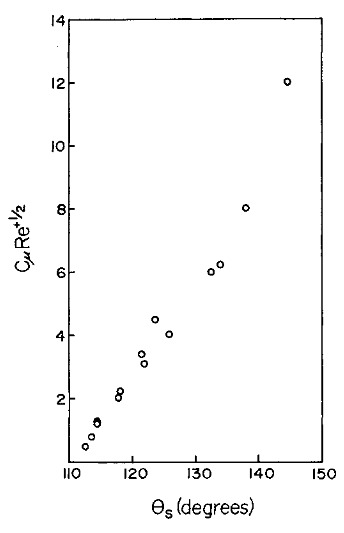

Fig. 3 Correlation of flow separation angle with jet momentum coefficient C_μ for moderate values of h/R.

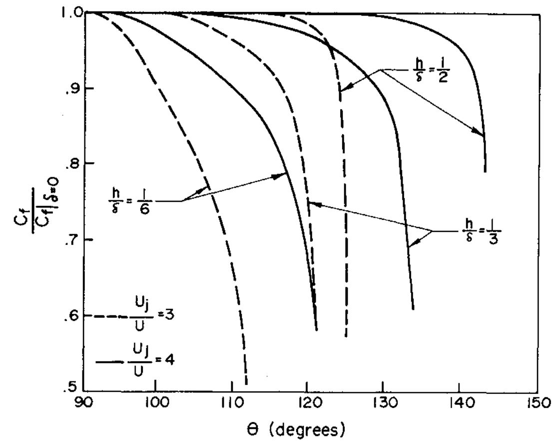

Fig. 4 Effect of upstream boundary layer on shear stress along the cylinder.

Circulation-Controlled Rotor

These results are now used to study the flow past a circular cylinder with circulation control. The approach was guided by the experimental observations of Dunham.[7] With the jet present, the pressure distribution is closely approximated by the potential flow result including circulation. The boundary-layer flow on the bottom surface is only weakly affected by the jet. Since the pressure on the cylinder in the separated region is essentially constant, the location of the separation point on the upper surface is completely determined from the bottom surface calculations. The forward stagnation point location is chosen (this fixes the circulation and thus the lift) and the jet strength is varied until the correct separation point is obtained.

The jet slot height h is chosen large enough so that U_j/U is the appropriate jet parameter and the effect of the upstream boundary layer is negligible. In Fig. 5, the lift coefficient $C_L = L/\rho U^2 R$ is given vs U_j/U for three slot locations. No experimental results are available for the laminar case so a comparison is made to the turbulent results of Cheeseman.[8] For these experiments, $Re = 4.5 \times 10^5$ and the Mach number is 0.3. $h/R = 0.024$ so that according to our theory and

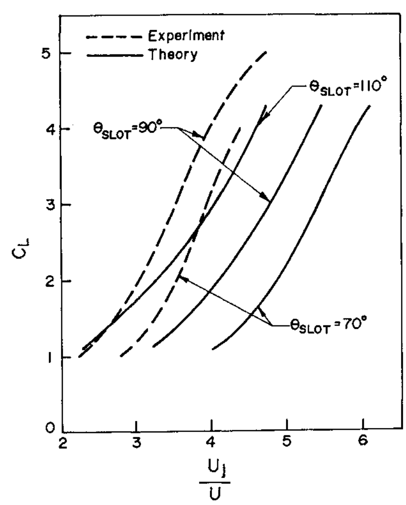

Fig. 5 Lift coefficient vs jet velocity ratio for circulation-controlled circular cylinder. Comparison with experiments (turbulent) of Cheeseman.[8]

Dunham's[7] observations, U_j/U is used in the comparison shown in Fig. 5.

Conclusions

The laminar theory shows that the momentum coefficient is an appropriate jet parameter for moderate values of h/R and U_j/U is appropriate for larger values. This is borne out by the experiments. The theory also shows that the effect of the upstream boundary layer is important for small h/δ and is most significant for smaller values of the jet speed. As the slot height approaches the boundary-layer thickness, this effect becomes negligible. The upstream boundary layer reduces the effectiveness of the jet in providing circulation control.

The theory shows also that the effectiveness of the jet is increased as the slot location is moved downstream. Dunham[7] warns, however, that this effectiveness is lost if the boundary layer on the cylinder separates before the slot is reached.

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Stress Redistribution and Instability of Rotating Beams and Disks

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Introduction

THIS Note reports the existence of a static inertio-elastic instability of rotating beams and disks that has been previously unnoticed. The instability is not the critical case (yielding occurs first) for rotating structures made of the usual metals but is of primary importance in rotating structures made of low-modulus, high-yield strength materials, such as some plastic materials. Hence, there may be applications of this work for example to tethered space stations and centrifugally stiffened plastic film helicopter rotors. The instability has been unnoticed because in previous analyses the centrifugal force has been assumed to be $m\Omega^2 r$ (for the beam) rather than the actual value of $m\Omega^2 (r + u)$, where m is the mass per unit length, Ω is the rotational speed, r is the initial undeformed radius, and u is the radial displacement. A corollary effect of this problem is that there is a stress-redistribution along the radius of the beam or disk due to the inclusion of the $\rho\Omega^2 u$ term at all values of Ω below the critical value. The analysis is given in the following two sections.

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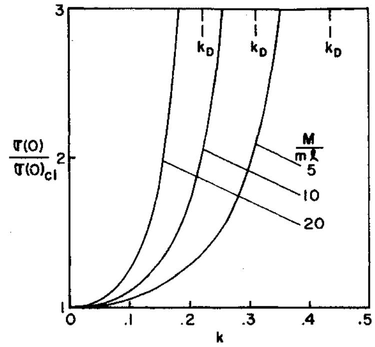

Fig. 1 Root stress ratio $\sigma(0)/\sigma\ (0)_{cl}$ vs rotational parameter.

Rotating Beam with Tip Mass

From elementary considerations the steady-state equation of motion is given by

$$m\Omega^2 l^2(u + l\xi) + (EAu')' = 0 \qquad (1)$$

where primes denote differentiation with respect to the nondimensional radial distance ξ(i.e., $0 \leq \xi \leq 1$). For a cantilevered beam with a tip mass M at the free end, the boundary conditions are $u(0) = 0$ and $u'(1) = (Ml/AE)\Omega^2[l + u(1)]$. Applications of the boundary conditions to the solution of Eq. (1) yields the results that the displacement and the stress are given by

$$u(\xi) = l(\{\sin k\xi/[k \cos k(1 - [M/ml]k \tan k)]\} - \xi) \qquad (2)$$

$$\sigma(\xi) = E(\{\cos k\xi/[\cos k(1 - [M/ml]k \tan k)]\} - 1) \qquad (3)$$

where

$$k^2 = ml^2\Omega^2/EA$$

The classical values of $u(\xi)$ and $\sigma(\xi)$, for comparison, are given by

$$u(\xi)_{cl} = k^2 l[M/ml(6 - k^2) + 3 - \xi^2]\xi/6 \qquad (4)$$

$$\sigma(\xi)_{cl} = Ek^2[M/ml(6 - k^2) + 3 - 3\xi^2]/6 \qquad (5)$$

It is noticed that Eqs. (2) and (3) possess instabilities when

$$k \tan k = ml/M \qquad (6)$$

and that Eqs. (4) and (5) possess no instabilities.

Table 1 K_D vs ml/M

ml/M	k_D
0	0
0.05	0.222
0.10	0.311
0.20	0.433
0.30	0.522
0.40	0.593
0.50	0.653
0.60	0.705
0.70	0.751
0.80	0.791
0.90	0.827
1.00	0.860
.	.
.	.
.	.
∞	1.571